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Comparison of Stable NLMF and NLMS Algorithms for Adaptive Noise Cancellation in ECG Signal with Gaussian, Binary and Uniform Signals As Inputs

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Abstract
The least mean fourth (LMF) algorithm has several stability problems. Its stability depends on the variance and distribution type of the adaptive filter input, the noise variance, and the initialization of filter weights. A global solution to these stability problems was presented recently for a normalized LMF (NLMF) algorithm. The analysis is done in context of adaptive noise cancellation with Gaussian, binary, and uniform desired signals. The analytical model is shown to accurately predict the optimum solutions. Comparisons of the NLMF and NLMS algorithms are then made for various parameter selections. It is then shown under what conditions the NLMF algorithm is superior to NLMS algorithm for adaptive noise cancelling.

Index Terms- Adaptive filtering, adaptive noise cancelling, least mean fourth algorithm, NLMS algorithm, and normalized least mean fourth algorithm.

I. INTRODUCTION

The least mean fourth (LMF) algorithm [1]-[17] outperforms well-known least mean square (LMS) algorithm [1], [2]. However, the LMF algorithm has several stability problems that may limit its applications. Reference [3] showed that the stability of the algorithm about wiener solution depends upon the adaptive filter input power and the noise power. References [4], [5], [7] showed that algorithm stability also depends on the initial value of the adaptive filter weights. References [8], [10], [11] showed that the LMF algorithm with unbounded regressors is not mean square stable even for the small values of the algorithm step-size. The above arguments suggest that LMF algorithm stability is more complicated than that of the LMS algorithm.

Normalized versions of the LMF algorithm have been studied [6], [9], [12]-[14] in order to improve performance. None of these normalized LMF (NLMF) algorithms provide a global remedy to the above mentioned stability problems. The weight vector update term of the LMF algorithm is normalized by squared norm of the regressor in [6], [12]. However, algorithm stability depends upon the input signal power [14]-[16]. The weight vector update term of the LMF algorithm in [9], [13] is normalized by a weighted sum of the squared norm of the regressor and the squared norm of the error vector. This algorithm was shown to diverge in [14]. An NLMF algorithm was proposed in [14], [15] with a weight vector update term that is normalized by the fourth power of the norm of the regressor. This normalization yields improvement in algorithm stability.

Finally a globally stable NLMF algorithm was presented in [17]. This algorithm is stable for all statistics of the input, noise, and weight initialization. The normalizing term is fourth order in the regressor and second order in the estimation error. The regressor normalization stabilizes the algorithm against increasing input power and the unboundedness of the input distribution. The estimation error normalization term stabilizes the algorithm against increasing noise power and increasing initial weight deviation. The algorithm is stable for all values of the step- size range between 0 and 2, a property similar to that of NLMS algorithm [1], [2]. However, the behavior of the globally stable NLMF algorithm is an extremely difficult task. This is primarily because the normalizing term is second order in the estimation error.

Thus, the present paper is primarily concerned with the globally stable NLMF algorithm. The model is useful since it provides conditions under which the new algorithm outperforms the NLMF algorithm. The results in this paper are the first to provide a model of its mean square performance. The analysis is done in the context of adaptive noise cancelling (ANC). Optimum solutions display excellent agreement with the theory, Finally the stable NLMF algorithm is compared with the NLMS algorithm and conditions are determined for ANC applications when one is preferred over other. For, example consider a pilot in an airplane. When the pilot speaks into a microphone, the engine noise in the cockpit combines with the noise signal. This additional noise makes the resultant signal heard by passengers of low quality information. The goal is to obtain a signal that contains the pilot voice but not the engine noise. Wecan cancel the noise with an adaptive filter if you obtain a sample of the engine noise and apply to adaptive filter.

The paper is organized as follows. Following the introduction, section II presents the Normalized LMS algorithm. Section III presents the stable NLMF algorithm. Section IV presents simulation results. Section V compares the proposed NLMF algorithm with NLMS algorithm Finally, conclusions are given in section VI.

II. NORMALIZED LMS ALGORITHM

Determining the upper bound step size is a problem for the variable step size algorithm if the input signal to the adaptive filter is non-stationary. The fastest convergence is achieved with the choice of step size as follows:

$$\mu_{max} = 1/(\lambda_{max} + \lambda_{min}) \tag{2.1}$$

The maximum step size in equation (2.1) does not always produce the stable and fast convergence, (2/3) μ_{max} is a rule of thumb for LMS algorithm. To increase the convergence speed Normalized LMS (NLMS) algorithm is a natural choice.

The NLMS is always the favorable choice of algorithm for fast convergence speed and for non-stationary input. The value of $\mu\sigma_x^2$ directly affects the convergence rate and stability of the LMS filter. In practice, the correction term applied to the estimated tap weight vector w (n) at the nth iteration is normalized with respect to squared Euclidean norm of the tap input x (n) at the (n-1)th iteration.

$$W(n+1) = W(n) + \frac{\alpha}{||x(n)||^2} e(n)x(n) \tag{2.2}$$

Apparently, the convergence rate of the NMLS algorithm is directly proportional to the NLMS adaptation constant α, i.e. the NLMS algorithm is independent of the input signal power. By choosing α so as to optimize the convergence rates of the algorithms, the NLMS algorithm converges more quickly than the LMS algorithm. It can also be stated that the NLMS is convergent in mean square if the adaptation constant is from 0 to 2 (however a more practical step size for NLMS is always less than unity) $0<\alpha<2$ Despite this particular edge that the NLMS exhibits, it does have a slight problem of its own. When the input vector x(n) is small, instability may occur since we are trying to perform numerical division by small value of the Euclidean Norm However, this can be easily overcome by appending a positive constant to the denominator in (2.2) such that

$$W(n+1) = W(n) + \frac{\alpha}{c+||x(n)||^2} e(n)x(n) \tag{2.3}$$

Where $c + ||x(n)||^2$ is the normalization factor. With this, more robust and reliable implementation of the NLMS algorithm is obtained.

A well-known tool that can be used to increase the stability of adaptive filtering algorithm is normalization. It is well known that the stability of the LMS algorithm is dependent of the input power of the adaptive filter. This makes it very hard, if not possible, to choose a step size that guarantees stability of the algorithm where there is lack of knowledge about the input power. This is solved by normalizing the weight vector update term$||x(n)||^2$. The resulting algorithm is termed as normalized LMS algorithm. This algorithm is stable for all input power, noise power, and the initial setting of adaptive filter weights, as long as step size is between 0 and 2.

For the case of stationary inputs, the cost function, also referred to as index of performance, is defined as the mean -square error i.e., the mean – square value of the difference between the desired response and filter output. The cost function is precisely a second-order function of the tap weights in the filter. The dependence of the mean-square error on the unknown tap weights may be viewed as a minimum point.

To develop a recursive algorithm for updating the tap weights of the adaptive filter, we proceed in two stages. First we use an iterative procedure to solve the equations; the iterative procedure is based on the method of steepest descent, which is a well-known technique in optimization theory.

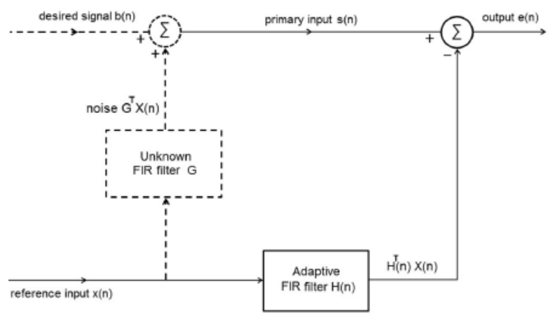


Fig. 1. Adaptive noise cancelling (the dotted parts are non-observable).

III. THE STABLE NLMF ALGORITHM.

Consider the case of ANC [1] shown in fig. The primary input to the canceller s(n) (also called plant output) is given by

$$s(n) = G^T X(n) + b(n) \tag{3.1}$$

where

$$G = (g_1, g_2 \, . \, . \, . \, . \, g_N)T \tag{3.2}$$

is the vector compose of plant parameters,
$X(n) = (x(n),\ x(n-1),\ .\ .\ .\ x(n-N+1)$
(3.3)

is the regressor vector at time n, where x(n) is the reference input, N is the number of plant parameters, b(n) is the desired signal and $(.)^T$ is the transpose of (.). The ANC is made by and adaptive FIR filter whose length is assumed equal to that of the plant. The adaptive filter provides an estimate of the noise $G^T X(n)$ corrupting the desired signal b(n). This estimate is subtracted from the canceller primary input to yield the canceller output e(n) given by

$$e(n) = s(n) - H^T(n)X(n) \qquad (3.4)$$

Where $H(n) = [h1(n), h2(n), .\ .\ .\ ., h_N(n)]^T$

is the weight vector of the adaptive filter. When the adaptive filter weights are near the plant parameters, the noise is cancelled and e(n) will be near the desired signal b(n).

The adaptation algorithm studied in this paper is the globally stable NLMF algorithm [17] defined by the stochastic recursion

$$H(n+1) = H(n) + \frac{\mu e^3(n)X(n)}{X^T(n)X(n)(X^T(n)X(n) + e^2(n))} \quad 0<\mu<2$$

(3.5)
where μ>0 is the algorithm step-size. Note that (3.5) behaves like the NLMS algorithm for large $e(n)$ and behaves like one form of the NLMF algorithm studied in [16] for small $e(n)$. This observation is important in the subsequent analysis.

The weight deviation vector is defined by

$$V(n) = H(n) - G$$

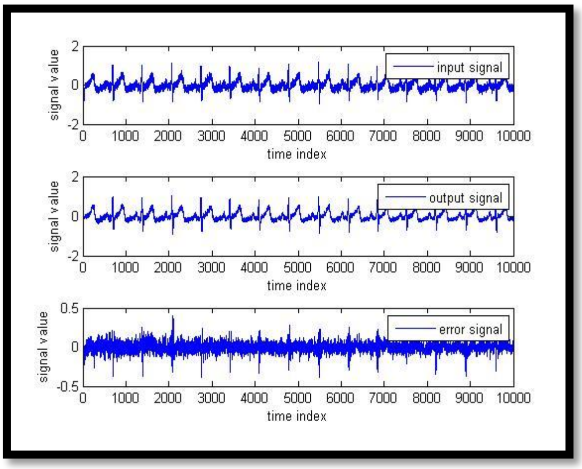
(3.6)
The instantaneous MSE is given by $E(V^T(n)V(n))$ where E denotes the mathematical expectation Due to (3.1), (3.4) and (3.6),

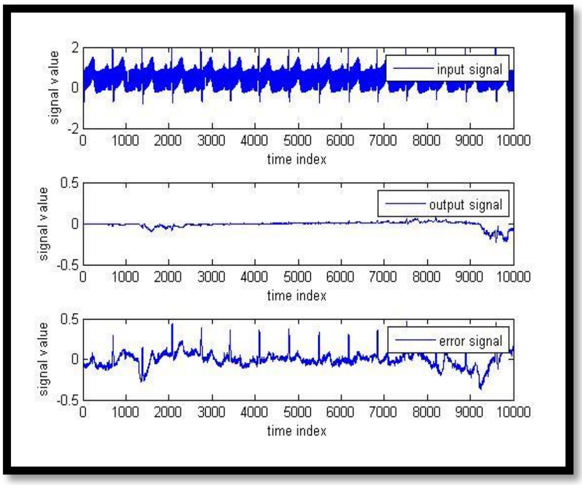
$$e(n) = b(n) - V^T(n)X(n)$$

(3.7)
The analysis of (3.5) is complicated by the presence of $e^2(n)$ in the denominator of the weight vector update term. Given this difficulty of the problem, the approximations made in this paper are reasonable. The proposed model helps users to decide if they should use the NLMF algorithm in a given application. This model will also help other researchers with new ideas for the analysis of nonlinear adaptive algorithms. The idea behind (3.5) is a combination of the ideas of the algorithms in [9] and [14]. The normalizing term in (3.5) (i.e. $X^T(n)X(n)(X^T(n)X(n) + e^2(n))$ is a fourth order polynomial in X(n). This term stabilizes the algorithm against the increase of the input variance since $e^3(n)X(n)$ is a fourth order polynomial in X(n). The normalizing term also stabilizes the algorithm for inputs with unbounded distributions, such as Gaussian inputs. The normalizing term in (3.5) also includes $e^2(n)$. This term stabilizes the algorithm against the increase of noise variance and the increases of the squared weight deviation, since $e(n) = b(n) - V^T(n)X(n)$.

IV. SIMULATION RESULTS OF NLMF AND NLMS ALGORITHMS

This section provides simulation results of NLMF and NLMS algorithms. The simulations are done for the case of adaptive noise cancelling with Gaussian, Binary and Uniform desired signals. The plant is a time-invariant FIR filter with equal parameters; $g_i = K$; i=1,2, . . . ,N. The initial weight vector of the adaptive filter is an all zero vector. The regressor vector is given by (3.3), where x(n) is the plant reference input.. K is selected much greater than one to yield a large initial weight deviation. This selection causes instability of the LMF algorithm and other NLMF algorithms. Thus, the globally stable NLMF algorithm is needed in this environment.





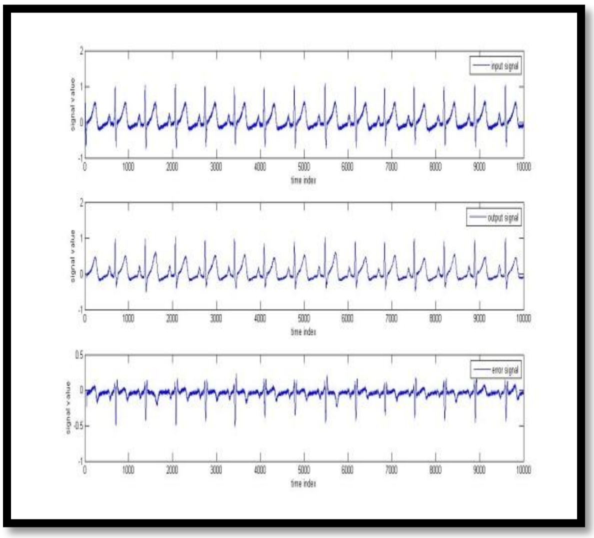


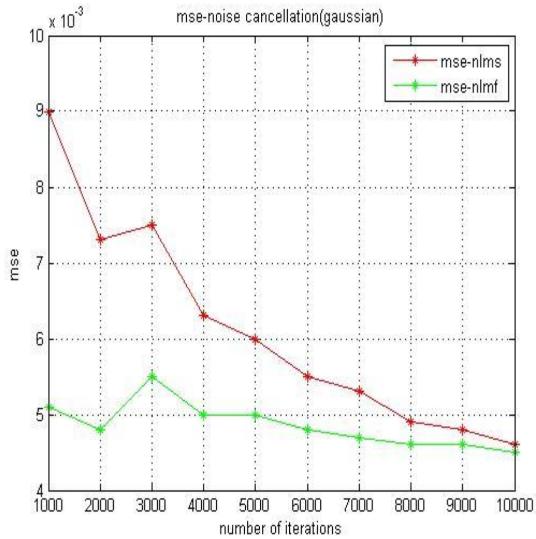
Fig 2: Simulated result of Noise Cancellation in ECG signals for a) Gaussian b) Binary c) Uniform using NLMF and NLMS algorithms.

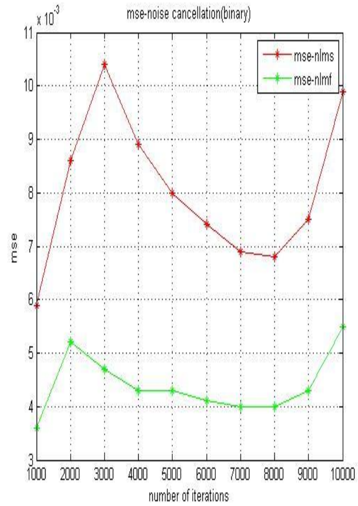
Iterations	Gaussian		Binary		Uniform	
	μ= 0.02 NLMS	μ=1 NLMF	μ= 0.02 NLMS	μ= 1 NLMF	μ= 0.02 NLMS	μ=1 NLMF
1000	.0090	.0051	.0060	.0043	.0120	.0092
2000	.0073	.0048	.0082	.0059	.0103	.0067
3000	.0075	.0055	.0103	.0049	.0099	.0066
4000	.0063	.0050	.0088	.0044	.0087	.0074
5000	.0060	.0050	.0081	.0043	.0084	.0073
6000	.0055	.0048	.0073	.0041	.0079	.0071
7000	.0053	.0047	.0069	.0039	.0069	.0078
8000	.0049	.0046	.0069	.0039	.0068	.0070
9000	.0048	.0046	.0073	.0043	.0069	.0072
10000	.0046	.0045	.0097	.0052	.0064	.0065

Table 1: MSE Comparison after Performing Various Number of Iterations in NLMS, NLMF for ECG (Gaussian, Binary and Uniform signals)

V. COMPARISONS OF NLMS AND NLMF ALGORITHMS

The NLMS algorithm is well-known and is often used as a benchmark for evaluating new adaptive algorithms. This section compares the MSE performance of the stable NLMF algorithm to that of the NLMS algorithm using simulations. The comparisons shown in figure 3 for the MSE behavior for each algorithm for $\mu_{NLMS} = 0.02, \mu_{NLMF} = 1$. The NLMS algorithm always converges quickly to (noise variable to input variable). This means that NLMS is preferred to NLMF for 0.02 as shown in figure 3 and the NLMF algorithm always converges quickly to (noise variable to input variable). This means NLMF is preferred to NLMS for 1. The following figure shows the mean-square performances of the ECG signal with Gaussian, Binary and Uniform desired signals. The graph is plotted between number of iterations versus the mean-square error.





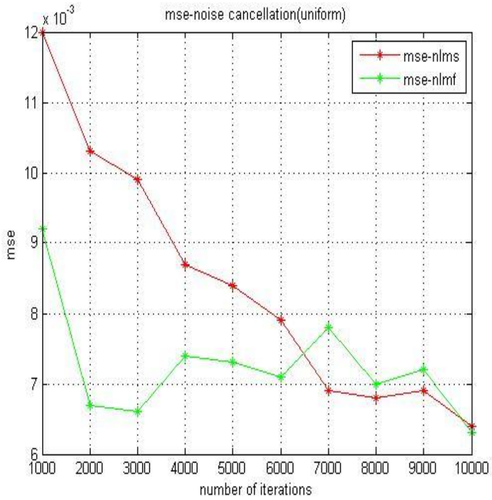


Fig 3: MSE (n) simulation results NLMS-(red), NLMF-(green) for $\mu_{NLMS} = 0.02$, $\mu_{NLMF} = 1$ (a) Gaussian (b) Binary (c) uniform.

VI. CONCLUSIONS

The Mean Square Error behavior of globally stable NLMF algorithm for Gaussian, binary and uniform signals for adaptive noise cancelling applications are shown. Optimum solutions show that analytical model accurately predicts the mean square error behavior. Comparisons of the globally stable NLMF and NLMS algorithms were made for various parameter selections. The NLMF algorithm is shown superior to NLMS algorithm for adaptive noise cancelling.

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